A STATISTICAL CONFRONTATION OF VOLUME & AREA IN NEW PLATONIC SOLID INDEX NUMBERS FOR A BETTER ESTIMATION OF GENERAL DEMOGRAPHIC (IM)BALANCE'S DYNAMICS [DEMO(GRA)STATISTICS AND DEMOGRAPHYSICS WITH VPSIN & APSIN]

Gheorghe Săvoiu

Romanian Statistical Society, Bucharest, e-mail: gsavoiu@yahoo.com

Abstract. *This paper aims at extending and deepening the understanding of the significance of the theoretical and pragmatic confrontation of two new statistical models of Index Numbers (hereafter: INs) in demography. The purpose of the article is to provide an example of the applied model at the demographic level that can and should be taken into account for increased quality of political, economic and social management decisions (hereafter: MD) aiming at ensuring the required efficiency and effectiveness at all three, from micro- to mezzo- and macro demographic levels. The originality of the Volume of Platonic Solid Index Number (hereafter: VPSIN) and Area of Platonic Solid Index Number (hereafter: APSIN) remains unbeatable, yet a statistical confrontation between VPSIN & APSIN, as well as between regular polyhedra inscribable in a sphere and regular polygons ("n-gons") inscribable in a circle at the same time, deepens the horizons of demographic research and improves the quality of political, economic and social decisions. A section of methodology and data clarifies the number of factors and the level of errors in the new models, implying a birth to the initial signs of a useful statistical confrontation simplified by an expression of the volumes, areas, versus type that has become standard in micro-, mezzo- and macro demography, including some new interdisciplinary science of demo(gra)statistics and demographysics for any managerial decisions: VPSIN vs. APSIN. The pragmatic structure of the paper contains a chapter dedicated to practical approaches through applied results and discussions about the validity and performance of both IN models as indices systems of factors in demographic policies of modern management. In a new proposed demo(gra)-statistics and a future demo(gra)physics context of some naturally dynamic imbalances, the statistical confrontation becomes a solid and originally determined path from a better demographic decision-making point of view with a great impact in political, economic and social management. The new instrumental abilities of VPSIN & APSIN are practically evaluated and many of the inquiring approaches finally allowed some decision-making hierarchies. This paper emphasized and can underline specific valences of the demo(gra)-statistics and demo(gra)physics as some new interdisciplinary sciences and valorizations of the IN functions. Final remarks recognize both, the advantages and disadvantages of VPSIN & APSIN, and identify several instrumental limits and perspectives of capitalization in contemporary demography, but especially in the future of political, economic and social management.*

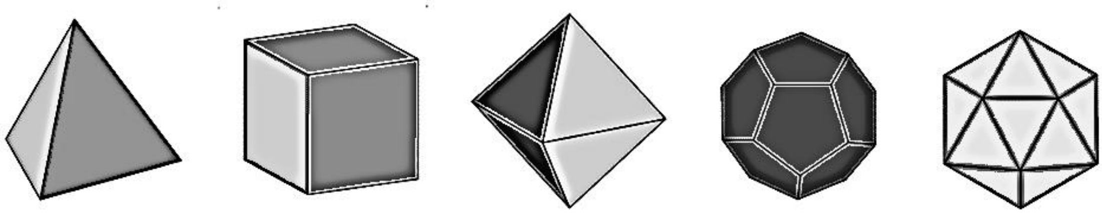
Keywords: *Euler's Polyhedral Formula (EPF); Geometric Area Index Number (GAIN); Index Number (IN); Index Numbers' Method (INM); Management Decision (MD); Demographic Imbalances Dynamics (DID); Platonic Solids (PS); Regular Polygons (n-gons); Regular Polyhedra (RPH); Volume & Area of Platonic Solid Index Number (VPSIN & APSIN); Demography; Demo(gra)statistics; Demographysics.*

1. INTRODUCTION

In the origin of this article, like in the history of Euclidean geometry, three names are closely related to regular convex polyhedra: Pythagoras, Plato and Archimedes. Of those three names, Plato's name, perhaps rather as the first parent of the first higher form of education and not necessarily as the great philosopher or mathematician, Plato was selected and attached to this pure geometric construction, both as volume and area, transforming them into Platonic solids [1,2]. A Platonic solid or regular polyhedron (RPH) can be defined mathematically as a three-dimensional solid that has identical faces generated by two-dimensional regular polygons. A mathematical demonstration identifies and limits the number of regular convex polyhedra to five usual types of "*polyhedra*": tetrahedron or pyramid, hexahedron or cube, octahedron or double pyramid, dodecahedron and icosahedron. In the famous Timaeus dialogue, Plato substituted the perfection and essence of this type of polyhedron (polyhedron) with the vital importance of the five elements of nature ([3]:

i) the tetrahedron with fire;
ii) the hexahedron with earth;
iii) the octahedron with air;
iv) the dodecahedron with ether;
v) the icosahedron with water.

There was an interesting and really scientific reason for Plato to choose the dodecahedron as the best representation of the whole universe. This reason can be a good cause even today, because geometrically the dodecahedron was and remains most closely approximates for the volume and area of a sphere, which was believed to be the shape of the universe at the time. Theaetetus, a contemporary of Plato, is the One who discovered the uniqueness of the five polyhedra, referring to the sum of less than 360^0 of the equal angles that meet in a polyhedron corner. The limit of 360^0 corresponded to exits from three-dimensional or geometric space and re-entry into two-dimensional or the plane of classical geometry. A polygon of 6 (six) or more 6 (six) sides naturally or Euclidean having an angle over 120^0 constituted the constructive limit, which naturally included only the equilateral triangle, the cube and the equilateral pentagon in the restrictive list of real constructive possibilities which they possess or lack are all explained according to their various purposes [4]. As Plato's Academy considered that the universe is the work of a Master who brought order to an initially disordered state of affairs, this article, and implicitly the research behind it, appeals to the uniqueness of the geometric (mathematical) model to explain not only the balances or imbalances of the classical Greek world but also those of the contemporary management of any economic entity, composed of fire (for visibility) and earth (for tangibility), but also of elements that ensure mediation like air and water, in a proportional progression ethereal to bind them together into a unitary, concordant whole [5]. Just as Euclidean geometry equated the shape of the body of the universe with a sphere, so the construction of an index number focused on Volume of Platonic Solids IN (hereafter: VPSIN) also starts from a sphere, being systematically confronted with another construction of an index number focused on Area of Platonic Solids IN (hereafter: APSIN derived from the creative idea of new geometric areas IN, known as GAIN) [6]. Figure 1 depicts the five usual types of "*polyhedra*" with the purpose to visualize similarities and differences between them, calling at the same time for conceptualization of a model for effective decision-making based on new statistical instruments & new economic symbols for water, ether, air, earth, and fire



Fire–Control	*Earth*–Production	*Air*–Stimulation	*Ether(Universe)*–Quality	*Water*–Demand
Tetrahedron	**Hexahedron**	**Octahedron**	**Dodecahedron**	**Icosahedron**

Source: Realised by the author.

Fig. 1. The five usual images of the "*regular polyhedra*" (RPH)

The foundation of effective management lies in effective decision-making (even in demographical cases), taking into account that management in a changing environment is profoundly based on everyday decision-making at strategic, tactical and operational levels. The highly fuzzy and volatile human environment put pressure on management to continuously revise and adopt management decision approaches and systems. Effective decision-making framework requires a holistic perspective, ambidexter view and incorporated change dynamism, considering external forces, internal development needs, and qualitative and quantitative tangible and intangible aspects of populations and individuals that influence the efficiency and effectiveness of management and decisions in general [7-10]. With that in mind, the key research question remains *which components or derived factors should be taken into account in a decision-making process, procedure, project, plan of action, programme, or scheme, designed for application in real-life practice, using the models based on the Index Numbers Method (hereafter: INM) and the New Index Numbers (INs) systems generated by VPSIN & APSIN?* There are also many key aspects that need to be discussed and clarified from a methodological point of view: i) *are all platonic solids useful indeed in the construction of the new indices or must exist an option for two or even only one* (choosing from icosahedron or dodecahedron, probably for their degree of proximity to the universal sphere of the investigated phenomenon and the appropriate managerial decision)?

The domain of IN or INs analysis must be completed here with good reasons to select from many detailed aspects regarding the mathematical (geometrical) way of calculating the volume and the area after changing the unit radius of the sphere in which the Platonic solid is inscribed (r_i), circumscribed (r_c) or even median (r_m), or regarding the groups or classes of IN extracted from the real phenomena (3-4 indices per social, economic, food, demographic field, etc.) or even connected to the aggregate INs calculus per domain and volume or area reconstruction depending on this similarity increase in volume and area (factors of equal importance) etc. Simultaneously, inside this paper, the statistical confrontation of some INs constructions (especially VPSIN & APSIN) must be realised and also some INs must solve the similitude to the number of phenomena's factors; ii) *is the new and optimal PSIN (VPSIN vs. APSIN) able to reflect a ratio between two platonic solids in the different units of time, space and even structure*? iii)*from a valid scientific test, how only one PSIN, chosen from VPSIN & APSIN, can offer a final quality of the construction after the confrontation* (new statistical-mathematical criteria are needed): by the level of error, the simplicity of the calculation, the complete statistical three-dimensional coverage (space, time and structure or from another criterion)? iv) *but the difficult task is not the calculus of the final volume or area, but the selection of the adequate objective method to choose the final applied aggregate* PSIN *(VPSIN vs. APSIN)?* The major research purpose of the article is to provide an example of the developed management model at the microeconomic level that can and should be taken into account for increased quality of management decisions (hereafter: MD) aiming at ensuring the required efficiency and effectiveness at both, microeconomic and macroeconomic levels. A statistical confrontation between VPSIN & APSIN, as well as between regular polyhedra inscribable in a sphere and regular polygons ("*n-gons*") inscribable in a circle at the same time, extends and deepens the horizons of managerial research and improves the quality of decisions.

The paper is structured as follows. The second section is dedicated to the literature review. The literature review unfolds the conceptual foundations of management, decision-making and the statistical index numbers method (INM). It exhibits background on the research problem and the significance of the new trend of perfection inside the management decisions based on new statistical instruments and new economic symbols for water, air, earth and fire. The research methodology is then introduced and includes the developed model for effective decision-making in the microeconomic area.

Afterwards, a section on the practical construction of VPSIN and APSIN was naturally required inside the article, the results and discussions of the study are also presented there. Finally, the paper concludes with the implications and recommendations for theory and practice, along with suggestions for future research. Some remarks of practically evaluated approach recognize both, advantages and disadvantages of PSIN and GAIN, and signify instrumental limitations and future perspectives of capitalization in contemporary management.

2. BRIEF LITERATURE REVIEW ABOUT NEW INTERDISCIPLINARY SCIENCES: DEMOGRA-STATISTICS OR DEMOGRAPHYSICS, AND SOME MAJOR THEORIES OF MANAGEMENT

Classic demography as an unidisciplinary science is no longer able to cover either traditional approaches or modern methods and techniques, not even accepting a lot of errors, trying to diminish all of these, from the calculus of their specific demographic indices, rates, quotas, weights in the classical and unilateral manner.

Analogous to classic demography, neither standard statistical investigations, nor even the holistic approaches, offered by the three-dimensional method of index number (temporal, spatial and structural) and, unfortunately, not even the science of (statistical) physics alone or applied individually can research, analyze and predict the complexity of the modern demography as an unidisciplinary or isolated science. Nowadays, none of the listed sciences is able to cover through the traditional diversity approaches in depth, to interpret without difficulty or from the classical and implicitly unilateral point of view the state, internal and external dynamics, explosion or implosion, rejuvenation or ageing, rurality or urbanity, the educational structure, the proportion of genders always changing or transgendering, the impact of the causes of death, the importance of the rate of fertility, the criteria structuring of the average life expectancy of human populations.

The first interdisciplinarity that is visibly created is placed at the intersection of demography and statistics, which can be defined as demographic statistics or demo(gra)statistics, with new accents generated by the creation of new indexes of greater complexity, but especially with physics (e.g. statistical physics), the last two or three sciences together becoming demographysics [11-13]. This literature confirms the need for an alternative interpretation of the demographical processes through statistics, physics, statistical physics and various subsystems that compose the demographic phenomena, such as migration or natural movement of human populations. Rigorous scientific research of the human population and of the global demographical system can be conferred by the vision of integrative interdisciplinarity offered through new sciences like demo(gra)statistics or demographysics, whose essential investigative approach or specific systems analysis is based on the premise that the properties do not reduce the number of individuals as units (atoms, particles, etc.). The keystone of the construction of the new systemic vision in the demo(gra)statistics or demographysics, developed in statistics for the first time, and subsequently in statistical physics starting using the existing deviations and variations in the area of statistical units. The same physical thought transforms the human populations in the studied objects and the system in interactions between subsystems using the probabilistic or stochastic nature of the behaviour units (as major components of the analyzed systems). The new approach of these interdisciplinary options is based first on statistics and demography [e.g. demo (gra)statistics as an intersection of these apparently independent sciences) and after this attempt on statistical physics and demography (e.g. demographysics as a reunion of both sciences).

As migration flows and natural movements represent the subsystems of the entire human beings system, so on all of these reunion aspects between the demographical theory and statistical or physical models become more adequate and comparable, sometimes even similar, to those conferred by statistics, statistical physics, and physics to sociology or economics, which have defined during the last three decades the new sciences called sociophysics and econophysics.

The demographysical science and its models are built in the same trans-, inter-, & multidisciplinary logic. Thus, the gravity or push-pull models in demography represented the beginnings of migration preoccupations, where the volume of a flow of international migration was defined as the result of the simultaneous action of the distance and the population of the two areas, that of origin and the intended destinations (i.e. John Quincy Stewart's model, introduced by the Princeton University astrophysicist in 1947-1948, "*focused on interdisciplinarity between demography and statistics or physics, based on a collection of repeatabilities and regularities, in the sense of demographic statistics*") [13]. New categories of the push-pull models included this major idea of identifying simplified perspectives and laws of demographic behaviour under the influence of the force of demographic attraction, demographic energy, demographic force of gravity, potential and gravity gradient, conceptualized by physics way and measured in a statistical manner by John Quincy Stewart. [i.e. the model of human gravity, well-known later as Carey-Stewart-Warntz model of the new school of quantitative geography, followed as importance based entirely on the Stewart major idea (Quincy, 1948; Garling, Golledge, 1993; Sen, Smith, 1995] [14-16].

In fact, demographic trans-, inter- and multi-disciplinary models are the real proof of the current and future delimitations of the development of the much-awaited and necessary sciences such as demo(gra)statistics and demographysics. This assertion is valid starting with the first model of Stewart and continuing with the relatively recent "*fractals*" models (Frankhauser, 1994), "*cluster & limited diffusion agglomeration* (C&ALD)" models (Gligor, 2012), or "*demographic implosion models by making use of cavitation*" (Săvoiu, Iorga, 2017) [17-19], The contributions of the author in the new domains of demo(gra)statistics or demographysics, were and still are influenced by specificity of statistical thinking, which is visible even in this paper dedicated to new geometrical index numbers, and by the hope of a necessary recognition, adequate to such a dynamic development,

The physical, statistical and mathematical laws, (i.e. especially geometrical figures, and solids with some regular area and volume in this paper) express not only the conservation of a quantity, but as well as the conservation of symmetries and kurtosis, or the space and time homogeneousness and structurality, and also the dynamic evolutions or involutions of populations' level.

In my opinion, the interdisciplinary needs of demography are placed inside the intersection with statistics, mathematics and physics, and seem to be nowadays the adequate expression of the accurate and integrated approach to the human population, through the reunited sphere of the preoccupations of these numerous sciences, which search to explain in a more and more various and detailed way, our human beings' dynamics, both in its quantitative side and, especially, in the qualitative one. The quality of the management decisions in modern demography, and also of the projections or forecasts essentially depends on the quality of the demo(gra)statistical data series processed with new demographysics methods and models, based on permanent new instruments and techniques [8]. The new inter-, trans-, and multidisciplinary sciences, succinctly described above, remains just modest expressions of the expansion of modern statistical thinking, of the passion for the essential details of statistical physics and of the contribution of physical thinking through the depths of the laws of physics, exploring together the universe of demography in general, from early interdisciplinary through trans-, and multidisciplinarity, to a really useful and necessary holistic approach.

Any synthesis of management significances tries to cover both their multitude and diversity and the continuous extension of the defined areas or of the approaches from the scholastic type (elevated and structuralized) to the procedural class (functional and interconnected) from the systemic attempts (disjoined into technological and human parts, but reunited afterwards predominantly attributively), to those of a chaotic [20] or random type (volatile and energy consuming through stability objectives) ending with those of a situational or contingent type (opportune and factually adequate), or to those based on total quality (Total Quality Management, whose only gift and generalized objective remains the quality) or the cultural ones (Cross-Cultural Management) [21].

Performance, development, continuous training, and knowledge have generated new practical methods, techniques & instruments of management, including in a perennial circuit, through theorizing and rendering essential on the level managerial models, subsequently validated or invalidated by resuming, a circuit specific to all sciences in general, although these exceed two thousand models, and many more thousands of techniques & instruments. Management theories represent a huge amount of different ideas, approaches, frameworks and guidelines with the purpose of better solving a vast variety of problems, from political to economic and social aspects (i.e. including demographical connections), which human societies have faced over their evolutionary and transformational journeys. However, the most prominent classification of management theories (MT) is recognized in three distinct and broad theories: i) classical or traditional theory; ii) neo-classical theory; iii) modern theory. Another approach distinguishes three other main categories altogether, respectively: a) technicist and nationalist economic theories; b) behavioural theories; c) cognitive ones [21-23]. In accordance with the first typology, management theories have evolved and multiplied over time and a brief presentation, naturally realised by the author in figure no. 2, tries to highlight their essences and differences.

I.CLASSICAL/TRADITIONAL MANAGEMENT THEORY
Born and evolved in the 19th century (eldest MT). **Major purpose**: To streamline work processes. **Included sub-theories**: Scientific MT, and Bureaucratic organization & administrative MT. **Basic (fundamental) ideas**: i) rigid hierarchical structure; ii) autocratic administration; iii) pyramid order; iv) strong control & command-based system. **Basic principle**: To focus on task efficiency & employees' motivation through economic & monetary rewards. **Out of date** in the contemporary world **Classic instruments:** methods, indicators, diagrams, etc
2. NEO-CLASSICAL MANAGEMENT THEORY
Born and evolved in Elton Mayo experiments (systemic MT) – 1924-1932. **Major purpose**: A company recognition as a social system. **Included sub-theories**: Human relations school, Maslow's hierarchy of needs, X-and-Y, Herzberg's two-factor theory, *behavioural school, Lateral process within the hierarchy.* **Basic ideas:** i) the birth of the humanised organization, ii) focused on the human orientation, iii) underlines behavioural aspect of employees, iv) drives forces of people; **Basic principle:** To revise the hierarchical structure of any organization and to point out that humans are the most valuable asset. **Out of date** in the contemporary world **Neo-classic instruments:** models, methods, techniques, factors, indicators, samples, graphs, etc
3.MODERN &POSTMODERN MANAGEMENT THEORY
Born and evolved from 1960 to the present (MT as an open system) **Major purpose:** A company is a distinct response to more and more complex external and internal factors, because "*one size does not fit all*" (keeping in mind systems, contingent approach, management science (MS) & organizational humanism at the core of organizational operations in a dynamic business environment). **Included sub-theories:** system theory (SMT), modern behavioural school (MBS), organizational humanism, contingency (OHCMT), operations management theory (OMT), management science (MST), contingency approach theory (CAMT) post-modern to modernity era, **Basic ideas:** i) MS is a quantitative approach to solving the various decision-making problems in organizations and societies; ii) MS develops mathematical models of distinct problems; iii) any construct of mathematically oriented models may be solved using various mathematical techniques; iv) MS encompasses a logical approach to problem-solving by applying mathematical models and computing technology & techniques; v) MS is a multidisciplinary field or domain for many related disciplines (e.g. mathematics, engineering, natural sciences statistics, informatics, econometrics, physics, etc.).. **Basic principle:** To redefine all management components of planning, organizing, commanding, coordinating and controlling into modern innovation, active organizing and encouraging, both entrepreneurship and self-control. **Up date**: uninterrupted trend in the contemporary world **Modern & postmodern instruments:** mathematical vision & engineering approach, complex statistics, physics & econometrics models based on more factors to solve less and less certain or stable variables and problems

Source: Realised by the author from [25-29].

Fig. 2. A brief presentation of major management theories (sub-theories) implied in economic, social and demographical evolutions

The new digital economy, the phenomenon of Big Data, robots and Chatbots, Machine Learning solutions, and especially the new Artificial Intelligence (AI) radically changed both the nature and the essence of human beings' associations and relationships, together with management science (MST) and artificial support for human intelligence. All of these transformed a traditional population into a more and more practical one, nearly eliminating the spatial or territorial constraints on human activities through a virtual contiguity process assuring a better communication bridge, and thus generating processing based on the systematic new spatial models associated with the combined time-territorial phenomenon of working online.

The first usual and generally acknowledged management methods remain [30]:

i) previsional management method (MP),
ii) method through objectives (MPO),
iii) method through exception (MPE),
iv) method through projects (MPP),
v) method on the product (MPPr),
vi) method through budgets (MPB),

vii) method through results (MPR),
viii method through systems (MPS),
ix) method through innovation (MPI),
x) collegial method (MC),
xi) method through consensus (MPC)
xii) specific statistical methods (SSM), etc.

Modern management science (MS), is more than a sub-theory of modern management theory (MST), being founded on an excessive quantitative approach combined with adequate psychological support to solve the various decision-making problems that confront management in any kind of organization and in human societies by developing statistical methods, including Index Number Method (INM) and its specific indicators, and mathematical models of all those problems.

3. METHODOLOGY & DATA, GEOMETRICAL RESEARCH METHODS FOR STATISTICAL CONFRONTATION

Despite numerous and distinctive approaches and factors that differ from one population to another, from an economic agent to another, from region to region or from market to market, a coherent managerial decision involves some basic criteria, sub-criteria, alternatives and, finally, all of these include systems of distinctive classic factors and even more neo-factors for constructing and developing an adequate structure inside statistical VPSIN & APSIN.

3.1. A general approach to a demographysics model based on the management decision theory

In the geometry of polyhedra, three types of radii related to the circumscribed (r_c), inscribed (r_i) and median (r_m) spheres are the most frequently used. In the case of the platonic solid type (regular polyhedron or RPH), the selection of the radius of the circumscribed sphere (r_c) ensures the maximum degree of variability of the final ratio that constitutes the volume or surface index (VPSIN or APSIN). An extensive variation and a maximum variability of the instrumental type in the case of the Index Numbers Method (INM) as in any other confrontation method, constitute perennial and valid statistical reasons for a better scientific measurement.

A significant methodological criterion for quantifying the variability of the investigated phenomena (demographic, economic, social, etc.) presupposes that each of the management decision factors is initially equal to a circumscribed sphere's radius of one unit (r = 1 or r = 100%).

A general management model is shown below in figure no. 3, to exemplify the complexity of the decision-making and the construct frame, including some major characteristics of demographic phenomenon.

External & Internal Factors	Management Decision Foundations (MDF)		Perspectives	Common goal
Political Legal Economic Demographics Socio-cultural Ecological Technological SWOT factors Searched advantages Studied disadvantages Risks vs opportunities Unknown errors' top Uncertainty level	Management functions applied in demography	Planning-Organizing-Leading-Staffing-Controlling (P-O-L-S-C)	Learning & growth Internal/external stakeholders Security& privacy Internal/external processes Organizational capacity Human-centric Organizational culture Population's traditions	Survival Health growth Sustainability Well-being Prosperity Reciprocity Balance Sense of good achievement Self content Satisfaction Trust
	Managers' Types & Roles in Organization (Population)	Decisional role + Interpersonal role + Informational role		
	Effective Management of Organizational Resources	Allocating resources: i) time; ii) people; iii) money; iv) assets		
	Applied Dimensions of Emotional and Partnership Intelligence (EQ & PQ)	a) self-awareness; b) social awareness c) self-management; d) social skills; e) partnership ability;		
	Know-how in Business development (surviving and improving life conditions for population)	Business goals' alignment with MDF (vision, mission, strategies, leadership, systems, structure, culture) + Strategist & operational levels' alignment		

Source: Realised by the author from [7-11 & 24, 31].

Fig. 3. A general approach to demographysics based on the management model as a foundation for decision-making construct

3.2. About the necessity of geometrical IN

Some methodological difficulties or problems can be eliminated or improved by the generation of new IN constructions like VPSIN & APSIN etc. These are specific geometric IN based on volume and area, focused on "*n-gons*", and on Platonic solids (especially on RPH) able to quantify the dynamics of complex phenomena and to meliorate many other aspects:

i) expanding the population of index numbers capable of correctly interpreting various other multidimensional markets, activities or fields under the impact of Big data or IoT - Internet of things (domestic robots market, suitability of managerial decisions for complex IT markets, etc.);

ii) the relative instrumental optimization and stabilization of the number of factors and their influence in the complex phenomenon analyzed dynamically or spatially and the constant updating and improvement of the sampling frame, etc.;

iii) solving through statistical confrontations the family of geometric indexes (VPSIN vs, APSIN) in the objective of spatial coverage, limitations and errors, in parallel with the identification and detailing of new weighting coefficients chained chronologically or spatially;

iv) the establishment of relevance limits or error thresholds with minimal anticipated effect in the annual change of the geometric indexes (surface, volume, etc.);

v) data optimization with regard to periodicity & non-periodicity, aggregation & disaggregation, elementary index chaining & *interruption* (aiming at the identification of n-gons and appropriate platonic solids, rather than artificial units);

vi) providing databases resulting from surface or volume geometric INs capable of leading to the calculation of elasticity, association, and correlation coefficients in the IN universe;

vii) based on modern statistical thinking, whose trends are increasingly approaching from inter- to trans-, & multidisciplinarity evolutions, a geometric IN (VPSIN or APSIN) becomes naturally a "*interpretive IN"* & a tool for regular confrontation with chronological & territorial coverage, adequate methodologies, with increasingly extensive and intensive quantification values, from the classic and economic area to the social, psychological, geographical, historical domains, etc (Săvoiu, 2015; Săvoiu *et al*, 2022) [6, 24].

The statistical confrontation method is a simple procedure of a strong practical purpose meant to validate a statistical solution, but, neither by conflict, nor by challenge, and only by a complete scientific dialogue or academic discussion about a statistical issue (variable, variability, instrument, method, methodology, etc.).Statistical confrontation involves opposing scientific viewpoints not only in time but also in space or even structurally, all of which are being viewed and finally considered just as trans-, inter- & multidisciplinary approaches.

"*Statistical thinking* [based on indices method respectively more precisely it means to interpret or analyze through quantitative values' interpretation with qualitative consequences the general level of an aggregate IN from the individual IN], *but, only hoping for ergodicity; ensuring not only stability, but also comparability, through confrontation method as a state of mind, visualizing, analysing and interpreting, in a manner that is not exclusivist or smooth, respectively uni- or two-dimensional, but rather, by extension, three-dimensional (simultaneously in a temporal, spatial and structural or organizational way)*" *(Savoiu, 2015, p. 16)* [24].

There are two mathematical notions in geometry such as polyhedron & polytope, based on different significances:

a) *polyhedron* is the generic notion of an object developed in any dimension;

b) *polytope* is just a bounded polyhedron and is more.

This paper used a generic notion or concept but with standard different dimensions of polytope generating multiple types such as nullitope (-1), moron (0), dion (1), polygon (2), polyhedron (3), polychoron (n) etc.

All the surfaces that delimit the Platonic solids are specific to three types of n-gons (where *n* is the number of sides or edges and n = 3, 4, 5) which have a special symmetry and equivalence. If a regular polygon is an n-sided or n-edged polygon (from the n-gons family) than the sides or the edges are all of the same length and are symmetrically placed about a common center and thus the polygon is both equiangular and equilateral. As a natural consequence, regular polyhedra (RPH) or Platonic solids (PS) are convex polyhedra and all possess the maximum symmetry, starting from three essential (regular) polygons: the equilateral triangle, the square and the pentagon [32-34].

A researcher can find an infinite number of regular polygons or n-gons, one for each positive number of sides or edges (*n*) such that $n \geq 3$, but the same researcher will find that this is not the case for the Platonic solids. RPH and especially Platonic solids are only five unique three-dimensional solids which consist of a collection of polygons joined at their edges, convex (the planes that bound the solids do not enter its interior and with all faces congruent regular polygons, and with the same number of faces at each vertex) (DeHovitz 2016) [34]. Starting from the usual elements of any regular polyhedral which are geometrical edges or sides (E), faces (F) and vertices (V), anyone can discover the combinatorial description of Schläfli symbolic or standard abbreviation where only sides or edges and faces are considered essential {n_S = the number of edges or sides surrounding each face, m_S = the number of faces, meeting at each vertex}. Two relations are important to validate the existence of only five Platonic solids. The first is purely geometric and underlines that by definition polyhedron is simply connected if every simple closed curve drawn on the surface can be shrunk to a point, and the second is known as Euler's Polyhedral Formula (EPF). (DeHovitz, 2016, p. 16) [34].

The derived equation of Euler theorem:

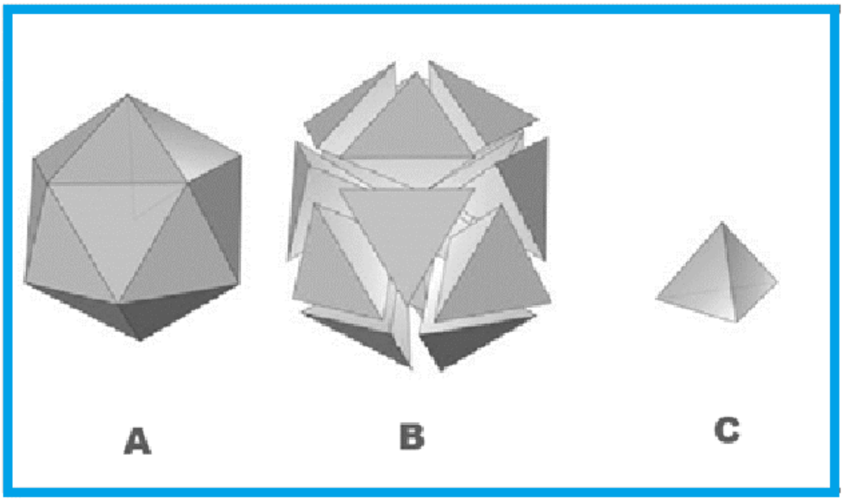
$$\mathbf{m \times V = 2E = n \times F} \tag{1}$$

leads to inequality:

$$\mathbf{4 > (n - 2)(m - 2)} \tag{2}$$

Knowing that n and m must each be at least 3, it can be seen that there are only 5 possibilities in Schläfli symbolic or standard abbreviation for {n, m}: {3, 3}, {4, 3}, {3, 4}, {5, 3}, {3, 5}.

Thus, this inequality has the well-known 5 ordered pair solutions that each corresponds to one of five Platonic solids. Finally, n and m become the limited values of n_S and m_S (3, 4, 5). Any of the Platonic Solids can be restructured or divided into equivalent or identical volume sub-components able to generate a stable number of substitutes or factors, identifying all sub-components (i.e. B from A) extracting one from the entire PS (i.e. C from A) in icosahedron's case as anyone can see in Figure no. 4:



Source: Realised by the author.

Fig. 4. The image of the entire icosahedron and the 20 equivalent or identical volume identical or standard subcomponents (substitutes or factors)

In fact, a statistical confrontation becomes necessary including not only a validated Euler formula but also identifying some substitutes identical to the final factors in the small RPHs' world able to generate PSIN, detailed through volume as VPSIN or area as APSIN (Table no. 1).

Table no. 1: Statistical confrontation based on vertices, edges (sides), faces and substitutes or factors

Regular Polyhedron (RPH)/Platonic Solid (PS) – using standard symbol Schläfli* {n_S, m_S}	Vertices (V)	Edges (sides) (E)	Feţe (F)	Validarea relaţiei lui Euclid V + F – E = 2	Number of identical volume subcomponents (substitutes or factors)
Tetrahedron {**3**, 3}	**4**	6	4	4 + 4 – 6 = 2	**4 with 3 - gon base**
Hexahedron {**4**, 3}	**8**	12	6	8 + 6 – 12 =2	**6 with 4- gon base**
Octahedron {**3**, 4}	**6**	12	8	6 + 8 – 12 = 2	**8 with 3 - gon base**
Dodecahedron {5, 3}	**12**	30	20	12 + 20 – 30 = 2	**12 with 5 - gon base** **60 with 3 - gon base**
Icosahedron {**3**, 5}	**20**	30	12	20 + 12 – 30 = 2	**20 with 3-gon base**

*Source: Realised by the author. *Note: RPH are identified by Schläfli standard symbol* {n_S, m_S}*, where n means the number of sides surrounding each face and m the number of faces that surround each vertex.*

There is an important aspect in dodecahedron's structure duality, based either on a 5-gon as essential area or face (e.g. dodecahedron is able to be split into 12 identical volume subcomponents or substitutes or factors based on 12 equivalent regular pentagons) but also on 3-gon (e.g. dodecahedron is able to be divided into 20 identical volume subcomponents or substitutes or factors based on 20 equivalent regular trigons). The tables below present the circumradius and the derived edge or side, length, volume and specific errors (Table no. 2a) and surface distinct errors (Table no. 2b), for each of the Platonic solids. The preference for the circumradius (r_c) from all the three possible values (r_c, r_m, r_i) is obvious from the calculus of statistical errors (Table no. 2a, and 2b).

The entire methodological analysis that follows is the result of Occam's razor-type logic, from the natural desire for simplification. In fact, the methodological attitude is a theatrical one in this case, the theatre performance or the theatrical act itself imposing a necessary densification of the space-time-conflict type, which facilitates or explains the scene of the decision itself. The conflicting states become the object of the managerial analysis of the decision-making factors or the determining indicators in the decision (macro or microeconomics).The confrontation is not purely mathematical or purely statistical [35-37], but hides behind planar or spatial geometry the intention to detect and optimize a system of decision indicators appropriate to the investigated phenomenon.

Table no. 2a: Errors' calculus for Circumscribed Sphere Volume (CSV)

PLATONIC SOLID (PS)	s =Platonic solid's edge or side (PSE) length based on circumradius equal to unity (r_c=1 or 100%)	PLATONIC SOLIDS' VOLUME(PSV)	- % from CSV Circumscribed Sphere's Volume**-	ERROR's LEVEL -%-
Tetrahedron	s=(r_c):[0.612372435] =1.6329931	V=[($\sqrt{2}$):12]×s³ = 0.5132002	12.2517523	> 87.74
Hexahedron	s=(r_c):[0.866025403] =1.1547005	V=[(s³)] = 1.5396007	36.7552593	> 63.24
Octahedron	s=(r_c):[0.707106781] = 1.4142136	V=[($\sqrt{2}$):3]×s³ = 1.3333333	31.8309878	> 68.16
Dodecahedron	s=(r_c):[1.401258538] = 0.7136442	V=[($15+7\sqrt{5}$):4]×s³= 2.7851639	66.4908903	= 33.51
Icosahedron	s=(r_c):[0.951056516] =1.0514622	V=[($15+5\sqrt{5}$):12]×s³= 2.5361507	60.5461381	= 39.45

*Source: Realised by the author. *Note: Value of* π *= 3.1415926536 and of circumradius (r_c) = 1 or 100%*

***Note: Circumscribed Sphere's Volume (CSV) = 4.1887902*

The decision to select a unique Platonic solid based on minimum error identifies in the dodecahedron the model of a system able to generate a minimum decisional error.

Table no. 2b: Errors' calculus for Circumscribed Sphere Surface (CSS)

PLATONIC SOLID (PS)	s =Platonic solid's edge or side (PSE) length based on circumradius equal to unity (r_c=1 or 100%)	PLATONIC SOLIDS' SURFACE (PSS) OR AREA (APS)	% from CSS - Circumscribed Sphere's Surface***	ERROR's LEVEL - % -
Tetrahedron	s = (r_c): [0.612372435] = 1.6329931	S=[($\sqrt{3}$) ×s²] = 4.6188022	36.7552601	> 63.2
Hexahedron	s = (r_c): [0.866025403] =1.1547005	S=[6×s²] = 8.0000000	63.6619773	> 36.3
Octahedron	s = (r_c): [0.707106781] =1.4142136	S=[$2\sqrt{3}$×s²] = 6.9282032	55.1328893	> 44.8
Dodecahedron	s = (r_c): [1.401258538] =0.7136442	S=[$3\sqrt{25+10\sqrt{5}}$]×s²= 10.5146222	83.6727049	= 16.3
Icosahedron	s = (r_c): [0.951056516] =1.0514622	S =[$5\sqrt{3}$×s²] = 9.5745414	76.1917796	= 23.8

*Source: Realised by the author. *Note: Value of* π *= 3.1415926536 and of circumradius (r_c) = 1 or 100%*

****Note: Circumscribed Sphere's Surface (CSS) = 12.5663706*

The methodological confrontation between revealing polyhedra and n-polygons for the optimization of the adequate selection of the indicator system (Table no. 3) leads to the same choice of the dodecahedron and the icosahedron as decision tools or solutions in management (micro or macroeconomic).

Table no. 3: Statistical confrontation based on edges and errors

Regular Polyhedron (RPH)/Platonic Solid (PS) - using standard symbol Schläfli**** {n_S, m_S}	Platonic solid's Factors (PSF)		"n-gons"	Polygons (n-gons) Factors (n-GF)		Differences (PSF) – (n-GF)	
	Edges (sides)	Errors		Edges (sides)	Errors	Edges (sides)	Errors
Tetrahedron {3, 3},	4	87.74	Trigon-Triangle	3	58.65	1	29.09
Hexahedron {4, 3},	8	63.24	Tetragon-Square	4	36.34	4	26.90
Octahedron {3, 4},	6	68.16	Hexagon	6	17.30	2	50.86
Dodecahedron {5, 3},	20	**33.51**	Dodecagon	12	4.52	12	28.99
Icosahedron {3, 5},	12	**39.45**	Icosagon	20	1.64	-8	37.81

*Source: Realised by the author. ****Note: RPH are identified by Schläfli standard symbol {n_S, m_S}, where n means the number of sides surrounding each face and m is the number of faces that surround each vertex.*

Finally, the methodology seeks to minimize the number of indicators selected by the system and correlated with the level of errors. Even the last confrontation of the last two Platonic solids with "*n-gons*" [6] leads to the selection of both constructions, dodecahedron and icosahedron for the level of errors (as minimum value) but also only to the icosahedron for the number of factors (as minimum value too).

The selection of *dodecagon* and *icosahedron* was caused not only by their duality but especially by the error level and a number of factors assimilated as the importance of complex phenomena. These two complex RPHs appear in biology (i.e. natural species well-known in scientific language as *Coccolithophore Braarudosphaera Bigelowii* and their *regular dodecahedral structure* & *Radiolarian Circogonia Icosahedra etc.).*

PSIN has many ways of calculus but the paper is structured in two manners (VPSIN and ASPIN) either as general formula or detailed to *dodecagon* and *icosahedron.* The starting point remains the ratio or the mathematical report in the simple way of thinking and evaluating, specific to statistical INM, used in the construction of any IN, including PSIN (VPSIN and ASPIN) and the value of circumradius (r_c) = 1 or 100%. A general PSIN's calculus is a statistical ratio or share between two volumes estimated in different units of time (t_0 and t_1), space (t_0 and t_1) or even structure (st_0 and st_1), following the volume variation (VPSIN) or the surface of the phenomenon (APSIN):

$$\text{VPSIN} = \frac{\text{Vi1}}{\text{Vi0}} = \frac{\sum_{i=1}^{n}(\text{Vi1})}{\sum_{i=1}^{n}(\text{Vi0})} \tag{3}$$

or

$$\text{VPSIN} = [\text{V}_{\text{i0}} + \textstyle\sum_{n=1}^{12}(\Delta V_n)] : \text{V}_{\text{i0}} \tag{4}$$

where Vi is the entire volume of the investigated phenomenon, estimated in different units 0 and 1, for any PS, based on the value of circumradius (r_c) = 1 or 100% for unit 0 and circumradius (r_c) variation for each factor's index for unit 1.

$$\text{APSIN} = \frac{\text{Ai1}}{\text{Ai0}} = \frac{\sum_{i=1}^{n}(\text{Ai1})}{\sum_{i=1}^{n}(\text{Ai0})} \tag{5}$$

or

$$\text{APSIN} = [\text{A}_{\text{i0}} + \textstyle\sum_{n=1}^{12}(\Delta A_n)] : \text{A}_{\text{i0}} \tag{6}$$

Obviously for any PS, PSIN's calculus (VPSIN and APSIN) can be detailed following the pure volume criterion (VPSIN) and the surface criterion of the phenomenon as convex volumetric coverage (APSIN). The number n is generated by PS, respectively it becomes the number of identical volume subcomponents (substitutes or factors) from the last statistical comparison column of table 1:

i) n = 4 with 3-gon base;
ii) n = 6 with 4-gon base;
iii) n = 8 with 3-gon base;
iv) n = 12 with 5-gon base / n = 60 with 3-gon base;
v) n = 20 with 3-gon base.

In this research, the author follows only the criterion of minimizing the statistical errors, and according to the data in tables 2a and 2b, the calculus was actually detailed exclusively for dodecahedron (1.e. n = 12 with 5-gon base) and icosahedron (i.e. n = 20 with 3-gon base) in mathematical relations {(7), (8), (9), (10)}.

First of all, any detailed PSIN's calculus (I - VPSIN and I - APSIN) describes the statistical-mathematical formulas that make the distinction in relation to classical indices and respects the conceptualization of the method (INM)

I – VPSIN (7)

$$= \frac{\sum_{i=1}^{n}(\text{Vi1})}{\sum_{i=1}^{n}(\text{Vi0})} = \frac{\sum_{i=1}^{n}([(15+7\sqrt{5}):4]) \times (\text{Si1})^3}{\sum_{i=1}^{n}([(15+7\sqrt{5}):4]) \times (\text{Si0})^3} = \frac{\sum_{i=1}^{n}(\text{Si1})^3}{\sum_{i=1}^{n}(\text{Si0})^3}$$

I – APSIN (8)

$$= \frac{\sum_{i=1}^{n}(\text{Ai1})}{\sum_{i=1}^{n}(\text{Ai0})} = \frac{\sum_{i=1}^{n}([3\sqrt{25+10\sqrt{5}}]) \times (\text{Si1})^2}{\sum_{i=1}^{n}([3\sqrt{25+10\sqrt{5}}]) \times (\text{Si0})^2} = \frac{\sum_{i=1}^{n}(\text{Si1})^2}{\sum_{i=1}^{n}(\text{Si0})^2}$$

where n = 12, s = Platonic solid's edge or side (PSE) length based on circumradius equal to unity in a standard case for IN calculus (r_c=1 or 100%) and s = (r_c): [1.401258538] =0.7136442.

I –VPSIN (9)

$$= \frac{\sum_{i=1}^{n}(\text{Vi1})}{\sum_{i=1}^{n}(\text{Vi0})} = \frac{\sum_{i=1}^{n}([(15+5\sqrt{5}):12]) \times (\text{Si1})^3}{\sum_{i=1}^{n}([(15+5\sqrt{5}):12]) \times (\text{Si0})^3} = \frac{\sum_{i=1}^{n}(\text{Si1})^3}{\sum_{i=1}^{n}(\text{Si0})^3}$$

I – APSIN (10)

$$= \frac{\sum_{i=1}^{n}(\text{Ai1})}{\sum_{i=1}^{n}(\text{Ai0})} = \frac{\sum_{i=1}^{n}([5\sqrt{3}]) \times (\text{Si1})^2}{\sum_{i=1}^{n}([5\sqrt{3}]) \times (\text{Si0})^2} = \frac{\sum_{i=1}^{n}(\text{Si1})^2}{\sum_{i=1}^{n}(\text{Si0})^2}$$

where n = 20, s = Platonic solid's edge or side (PSE) length based on circumradius equal to unity in a standard case for IN calculus (r_c=1 or 100%) and s = (r_c) : [0.951056516] = 1.0514622.

Exemplifying with the help of 12 and 20 factorial index values according to Tables 4a, 4b and 4c, and in parallel 5a, 5b and 5c (in the second case of icosahedron keeping unchanged the first of the initial set of 12 values and completing it with another 8 values). Some statistical advantages of PSIN (VPSIN and APSIN) are obtained based on their specific graphic expressions from the Figure 4 and 5, completed by the normal distributions (Figure 5 and 6). Some significant differences to the extreme values are confirmed in the cases of the distinct volumetric and surface developments similar to mathematical functions $f(x^2)$ and $f(x^3)$ types of the new detailed PSIN, where x is assimilated to a classic IN [38-41].

Volume's changes are visible after circumradius is modified for each of 12 substitutes or factors transformed in statistical indicators (indices) in dodecahedron case:

Table no. 4a: Statistical confrontation of classic IN, VPSIN and APSIN
(based on the aggregate of volumes and areas in calculus in dodecahedron)

Classic calculus of IN *Standard value IN = 1.000*	**Volume calculus (VPSIN)** *Standard value for IN =1 and (r_c) = 1* $V=[(15+7\sqrt{5}):4]\times s^3$= 2.7851639	**Area calculus (APSIN)** *Standard value for IN =1 and (r_c) = 1* $S=[3\sqrt{25+10\sqrt{5}}]\times s^2$= 10.5146222
0.902	2.0439504546	8.5547387149
0.941	2.3207033114	9.3104982178
0.970	2.5419438584	9.8932080679
0.987	2.6779484315	10.2430190352
1.000	2.7851638631	10.5146222424
1.001	2.7935277130	10.5356620015
1.030	3.0434237527	11.1549627369
1.050	3.224175317	11.5923710222
1.068	3.392848829	11.9932304806
1.075	3.4600003635	12.1509603289
1.081	3.5182592124	12.2869774822
1.100	3.7070531018	12.7226929133
12.205 : 12 = **1.0170833333**	35.5089982084 : 12=2.959083183 2.959083183 : 2.7851638631= **1.062444915**	130.9529432438 : 12 = 10.91274527 10.91274527 : 10.5146222424 = **1.03786375**

Source: Realised by the author

*Note: Value of π = 3.1415926536 and of circumradius (r_c) = 1 or 100%

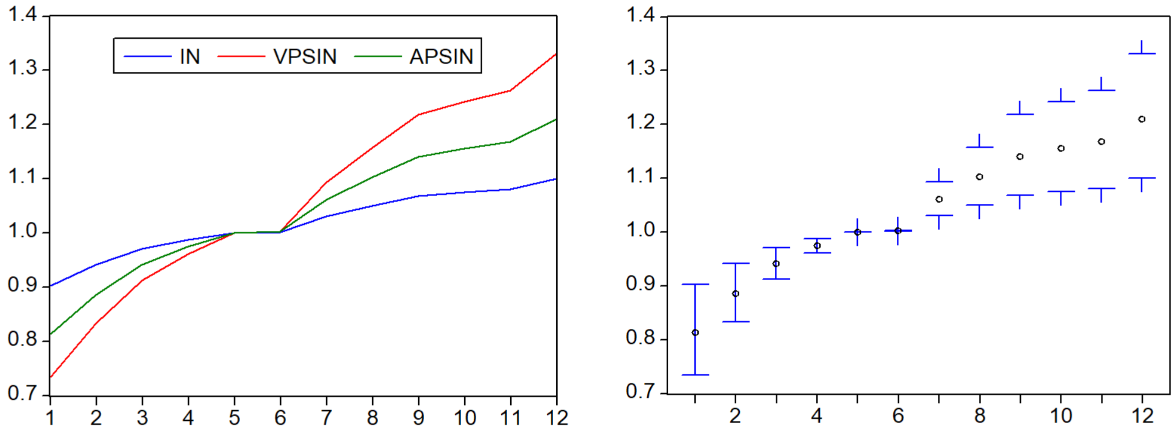
Table no. 4b: Statistical confrontation of classic IN, VPSIN and APSIN
(based on direct indices of volume and area in dodecahedron)

Classic calculus of IN *Standard value IN = 1.000*	**VPSIN** *Standard value for IN =1 and (r_c) = 1*	**APSIN** *Standard value for IN =1 and (r_c) = 1*
0.902	0.7338708082	0.8136039946
0.941	0.8332376213	0.8854809941
0.970	0.9126730003	0.9408999938
0.987	0.9615048033	0.9741689935
1.000	1.00000000000	0.9999999934
1.001	1.0030030014	1.0020009934
1.030	1.0927270004	1.0608999930
1.050	1.1576250004	1.1024999927
1.068	1.2181864324	1.1406239925
1.075	1.2422968754	1.1556249924
1.081	1.2632144414	1.1685609923
1.100	1.3310000005	1.2099999920
12.205 : 12 = **1.01708333**	12.749338985 :12 = **1.062444915**	12.4543649177 : 12 = **1.03786375**

Source: Realised by the author

* Note: Value of π = 3.1415926536 and of circumradius (r_c) = 1 or 100%

** Note: The major statistical advantage of PSIN (VPSIN and APSIN) is offered in the comparison in Table 4b by the extensive variation of the values of the new volume and area indices in the dodecahedron case.



Source: Realised by author based on data from table 4b

Fig. 5. Visual differences between classic IN and PSIN (VPSIN and APSIN) as the extended variation of the extreme or limiting values in the dodecahedron case

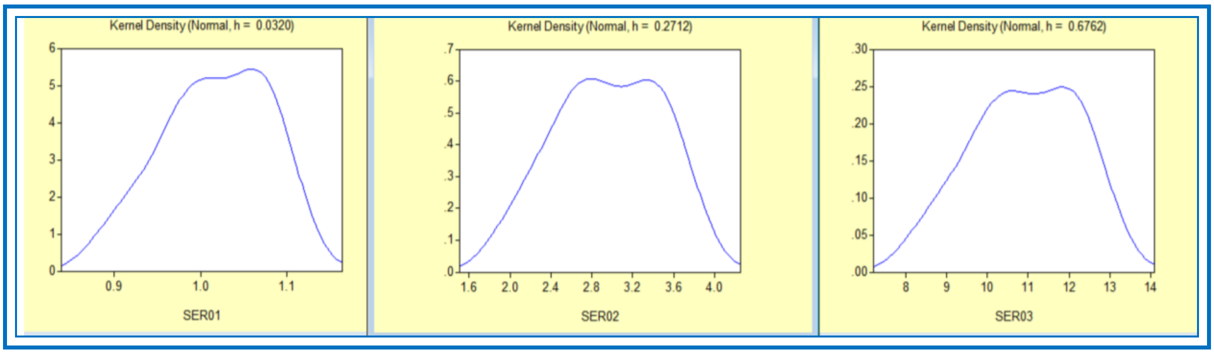
PSIN (VPSIN and APSIN) values are still highly correlated on a level relatively identical to classic IN according to the Correlation Matrix based on data from Table 4b:

Table no. 4c: Correlation Matrix between IN, VPSIN and APSIN in dodecahedron case

	IN	VPSIN	APSIN
IN	1.000000	0.998357	0.999582
VPSIN	0.998357	1.000000	0.999597
APSIN	0.999582	0.999597	1.000000

Source: Realised by the author based on data from the Table 4b

Even Kernel distributions are relatively similar for the three series of data (classic IN, VPSIN and APSIN (Figure no. 6):



Source: Realised by the authors based on data from Tables 4a

Fig. 6. Visual relative identity between classic IN and PSIN (VPSIN and APSIN) as Kernel distribution in dodecahedron case

Table no. 5a: Statistical confrontation of classic IN, VPSIN and APSIN based calculus in Statistical confrontation of classic IN, VPSIN and APSIN
(based on the aggregate of volumes and areas in calculus in the icosahedron)

Classic calculus of IN *Standard value* *IN = 1.000*	**Volume calculus (VPSIN)** *Standard value for IN =1 and (r_c) = 1* $V=[(15+5\sqrt{5}):12]\times s^3$ = 2.5361507101	**Area calculus (APSIN)** *Standard value for IN =1 and (r_c) = 1* $S=[5\sqrt{3}\times s^2]$ = 9.5745413833
0.900	1.8488538677	7.7553785205
0.902	1.8612069708	7.7898851676
0.925	2.0072444050	8.1922169711
0.941	2.1132161842	8.4780744786
0.950	2.1744322151	8.6410235984
0.965	2.2790665020	8.9160522996
0.970	2.3146762771	9.0086859875
0.975	2.3506550621	9.1017984025
0.980	2.3870047592	9.1953895445
0.987	2.4385210889	9.3272214048
1.000	2.5361507101	9.5745413833
1.001	2.5437667732	9.5937000406
1.030	2.7713203570	10.1576309535
1.040	2.8528246324	10.3558239601
1.050	2.9359114658	10.5559318751
1.056	2.9865295239	10.6769157800
1.068	3.0895043846	10.9209516908
1.075	3.1506521017	11.0645793860
1.081	3.2037022016	11.1884356534
1.100	3.3756165952	11.5851950738
19.996 : 20 = **0.9998**	51.2208560776 : 20 = 2.561042804 2.561042804:2.5361507101= **1.0098149**	192.0794321717 : 20 = 9.603971605 9.603971605: 9.5745413833 = **1.0030738**

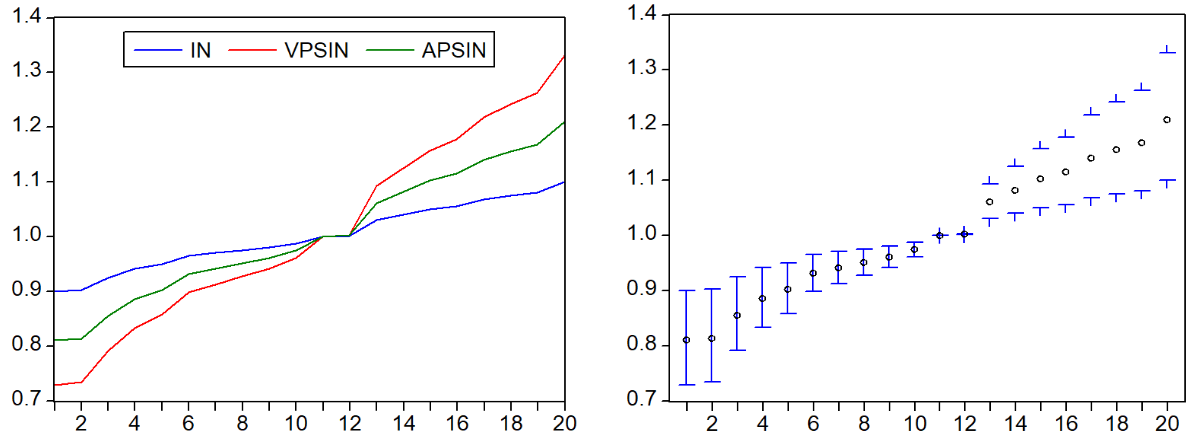
Source: Realised by the author. *Note: Value of π = 3.1415926536 and of the circumradius (r_c) = 1 or 100%

Table no. 5b: Statistical confrontation of classic IN, VPSIN and APSIN
(based on direct indices of volume and area in icosahedron)

Classic calculus of IN *Standard value IN = 1.000*	**VPSIN** *Standard value for IN =1 and (r_c) = 1*	**APSIN** *Standard value for IN =1 and (r_c) = 1*
0.900	0.7289999998	0.8099999933
0.902	0.7338708078	0.8136039932
0.925	0.7914531248	0.8556249929
0.941	0.8332376208	0.8854809926
0.950	0.8573749998	0.9024999925
0.965	0.8986321248	0.9312249923
0.970	0.9126729998	0.9408999922
0.975	0.9268593748	0.9506249921
0.980	0.9411919998	0.9603999920
0.987	0.9615048028	0.9741689919
1.000	0.9999999998	0.9999999917
1.001	1.0030030008	1.0020009917
1.030	1.0927269997	1.0608999912
1.040	1.1248639997	1.0815999910
1.050	1.1576249997	1.1024999909
1.056	1.1775836157	1.1151359907
1.068	1.2181864317	1.1406239905
1.075	1.2422968747	1.1556249904
1.081	1.2632144407	1.1685609903
1.100	1.3309999997	1.2099999900
19.996 : 20 = **0.9998**	20.1962982172 : 20 = **1.0098149**	20.0614758334 : 20 = **1.0030738**

Source: Realised by the author *Note: Value of π = 3.1415926536 and of circumradius (r_c) = 1 or 100%

The major statistical advantage of PSIN (VPSIN and APSIN) is also maintained in the case of the icosahedron and it is offered in the comparison in table 5b by the extensive variation of the values of the new volume and area indices.



Source: Realised by the author based on data from Table 5b

Fig. 7. Extended variation of the extreme or limiting values underlined from visual differences between classic IN and PSIN (VPSIN and APSIN) in icosahedron case

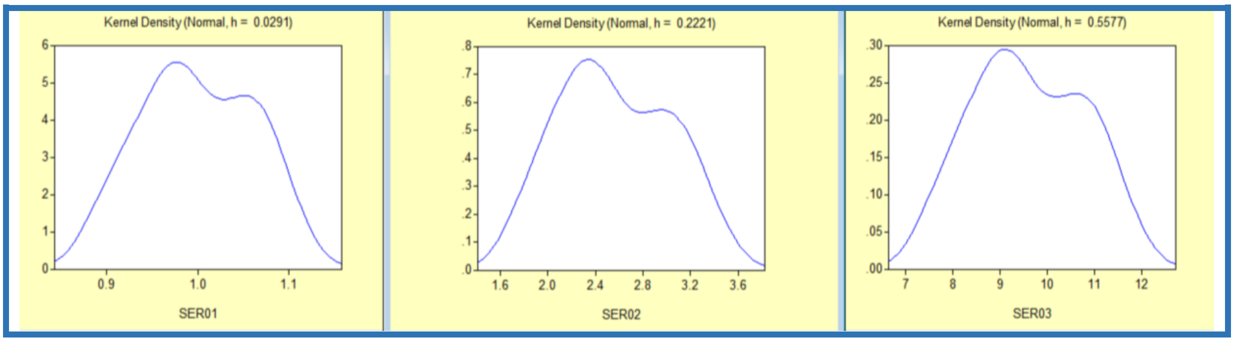
In the icosahedron case, PSIN (VPSIN and APSIN) values remain also highly correlated on a level relatively identical to classic IN according to the Correlation Matrix based on data from Table 5b:

Table no. 5c: Correlation Matrix between IN, VPSIN and APSIN in the icosahedron case

	IN	VPSIN	APSIN
IN	1.000000	0.998442	0.999605
VPSIN	0.998442	1.000000	0.999615
APSIN	0.999605	0.999615	1.000000

Source: Realised by the author based on data from Table 5b

Kernel distributions remains also relative similar for the three series of data (classic IN, VPSIN and APSIN (Figure no. 8) and in the icosahedron case:



Source: Realised by the author based on data from Tables 5a

Fig. 8. Visual relative identity between classic IN and PSIN (VPSIN and APSIN) as Kernel distribution in the icosahedron case

In conformity with Kernel Density and its formal graph (assuming a normal distribution of the PSIN data (VPSIN and APSIN) does not reveal disadvantages either apparent or of statistical substance or in-depth in relation to classic IN, either for the dodecahedron or for the icosahedron. Dodecahedron and Icosahedron cases have some major advantages from confrontation with other PS and these aspects refer to: a minimum level of error, an optimal number of substitutes or factors in the investigated phenomenon, and an extended variation of individual IN values from PSIN.

These examples and the entire methodological section underline the advantages of PSIN (VPSIN and APSIN) confronted with the classical IN in the dodecahedron. In fact, the advantages are really more visible than disadvantages, especially in the case of dual factors of the dodecahedron (12 or 60 subcomponents) and the icosahedron (20) not only as lower error level from the statistical confrontation with volume of circumscribed sphere in VPSIN calculus (33.51% from Table 2a) and area of circumscribed sphere in APSIN calculus (16.3 % from Table 2b), but also as smaller number of substitutes or factors in the investigated phenomenon and as the biggest variation of the factorial indices, etc.

4. RESULTS AND DISCUSSIONS

Any time when a "demographic change" appears also it requires not only verified measurements and an aggregate vision but also some urgent economic & social policy developments. Whenever a "major demographic change" appears it also requires not only new and profoundly verified measurements, together with an aggregate vision based on a relevant selection of some demographical index numbers (INs), but also a synthetic indicator about the population's evolution (involution). This new type of IN must be able to characterize demographic trends as a whole, starting from a reasonable number of constructive details focused on essential sub-phenomena and allowing the identification of urgent economic & social policy solutions based on new developments.

Everything becomes similar to the understanding of the impact between two populations in time and space, but even structurally, like a collision of an asteroid with the Earth based on a single final indicator to bring together all the existing speeds, from those related to opposite impulses to those of mutual attraction and repulsion, from gravitational ones to anti-gravitational ones, etc. Such a multidisciplinary approach with holistic tendencies [42-43] can by extension ensure a comparability of all types of velocities or dynamics that appear, from areolar to angular velocity, from orbital to gravitational velocity, from absolute to relative velocity, from the speed of attraction to the speed of repulsion, etc. Classical demographic dynamics require an aggregate index that, by analogy with Kepler, validates the third law of planetary motion, where "*the square of the period of the planet's revolution should be in a relatively proportional relationship with the cube of the semi-major axis of the orbit*" [44-45]. This first physical aspect leads to a demographic type of thinking that measures, correlates and even confronts the evolution of multiple dimensions and speeds in geometric terms of surface or area and volume or space. In this stage, prior to a more or less generalized space-time relativization, the new geometric index, proposed in this article and entitled PSIN (APSIN from area or VPSIN from volume) can be constructed with the help of either only 12 factors in the dodecahedron or another 20 factors in icosahedron where partial or individual index numbers (visible images as factorial statistical variables) describe the very different ratios of the speeds of fertility, death, birth, migration (immigration and emigration), rejuvenation or ageing, nuptiality or divorce, illiteracy or education, etc. How could these connections between demography speeds as demographical phenomena measured through individual indices influence the balance of the human universe, finally quantified through an aggregate index number as a result? The way in which these influences, associations and interconnections between the different demographic speeds could influence the balance of the human universe can be given as an example of the complexity of the relationships (i.e. the speed of ageing changes productivity and well-being, ultimately influencing the speed of population growth, by increasing emigration or by reducing demographic nuptials).

The initial vision is statistical and the final justifying one becomes physical and the jump from classical demography to demo(gra)statistics and finally to demography becomes a necessary and much easier to understand transformation in the dynamics of human populations or even in abstract populations.

The new instruments' application in a new domain of demo(gra)statistics or demographysics must be followed by a necessary statistical confrontation, and this research paper does it consequently. In a natural practice of the new geometric indices proposed from the two PSIN cases (VPSIN and APSIN) and in order to facilitate the management requirements and make the appropriate or optimal decisions, both political, economic, social and cultural related to the demography of an extended area (i.e. the European Union) implies first of all, the identification of a set of derived demographic phenomena that must contain both 12 and 20 key decisional factors, respectively the use of a dodecahedron (12 factors) or icosahedron (20 factors) PSIN type index (VPSIN or APSIN).

In this specific way of demo(gra)statistical or demographic physical thinking or in this more profound and distinctive sense, tables no. 6 and no. 7 represent the selected factors and together with 6a, 6b, and 7a, 7b, offer a demographic PSIN calculus and statistical comparison. In another typically statistical way of thinking and parsimony-focused vein, all calculations as referred to all VPSIN and APSIN indicators in a dodecahedron & icosahedron as major hypotheses, based on 12 and 20 factors transformed into classical INs are similarly resumed. The final aggregated value of such an original Index Number, based on Platonic Solids (PSIN), express naturally in a geometric manner, more complex & more truthful in relation to the methods, techniques and instruments focused on classical indices (i.e. classical Index Number Theory -INT & classical results, known as Index Numbers-INs).

Table no. 6: Some Major Demographical Factors (generating the 12 PSIN's key factors in EU)

Previous year = 1.000000

No	A key factor and derived Index Number	Value*
1.	Deaths (2020/2021)	0.9786276
	5,184,078 : 5,297,294 = 0.978627578	
2.	Life births (2021/2020)	1.0041778
	4,088,494 : 4,071,484 = 1.004177838	
3.	Net natural change (2021/2020)	0.9969231
	(0.972) : (0.975) = 0.996923076	
4.	Infant mortality rate (2021/2020)	0.9762795
	3.14568 3.22211 : = 0.976279518	
5.	*Immigration from outside EU (2021/2020)*	1.1760124
	2,255,406 : 1,917,842 = 1.176012414	
6.	Emigration to outside EU (2020/2021)	0.8570580
	956,247 : 1,115,732 = 0.857057967	
7.	Healthy life years at birth (2021/2020)	0.9907121
	64.0 : 64.6 = 0.990712074	
8.	Fertility rate (2021/2020)	1.02
	1.53 : 1.50 = 1.02	
9.	Ageing (65 years & more) (2020/2021)	0.9855769
	0.205 : 0.208 = 0.985576923	
10.	Crude divorce rate (2020/2021)**	0.9657647
	1.6:1.7 = 0.94117647+final correction = 0.965764705	
11.	*Crude marriage rate (2021/2020)***	1.1273125
	3.9 : 3.2 = 1.21875 + final correction =1.1273125	
12.	Life expectancy at birth (2021/2020)	0.9962627
	80.1: 80.4 = 0.99626268656	

Source: Realised by the author from available online data at: https://ec.europa.eu/eurostat/databrowser/ product/view/

*All of the demographic key factors' information is selected from https://chat.openai.com being analyzed by the author based on the criterion of a maximum of 12 dodecahedron's indicators, and thus becoming a narrow list of a more extensive one in relation to the data availability. The data construction of comparable indicators is based on both statistical and demographic standards. *Note: Each value is expressed as index numbers (IN) according to the positive impact of the key factor on the demographic evolution of the EU, dividing the indicator from 2021 to that of 2020 (i.e. births) or vice versa (i.e. deaths). **Note: 58.2 % of children are born inside marriages two corrections were needed to IN from lines 11-12.*

Once the 12 significant indicators have been selected (Table no. 6) that describe, in the spirit of the classic number index, the relationship between various fertility rates, natural movement, migratory movement, life expectancy (including healthy life expectancy), urbanization, ageing speeds, divorce and nuptiality, as well as speeds or accelerations related to structural changes and the replacement of some age structures with others etc. a demographic-type geometric index can be determined, which is a final aggregate of these demographic speeds according to the examples in the methodological section.

Table no. 6a: Statistical confrontation of the demographic IN and PSIN (VPSIN & APSIN), based on aggregates of volumes & areas, through a pragmatic calculation focused on a dodecahedron

IN Classic calculus *Standard value IN = 1.000*	**Volume calculus (VPSIN)** *Standard values IN =1 &(r_c) = 1* V=[$(15+7\sqrt{5})$:4] $\times s^3$ = 2.7851639	**Area calculus (APSIN)** *Standard values IN =1& (r_c) = 1* S=[$3\sqrt{25+10\sqrt{5}}$]$\times s^2$ = 10.5146222
0.9786276	2.6103763817	10.0699796813
1.0041778	2.8202174758	10.6026617423
0.9969231	2.7595338740	10.4500169053
0.9762795	2.5916315709	10.0217142282
1.1760124	4.5298774688	14.5417768688
0.8570580	1.7534015694	7.7234991063
0.9907121	2.7082774485	10.3202117673
1.02	2.9556381729	10.9394129810
0.9855769	2.6663815738	10.2135026591
0.9657647	2.5087924474	9.8070034723
1.1273125	3.9900994434	13.362333898
0.9962627	2.7540534436	10.4361765090
12.074707: 12=**1.0062 256**	(34.6482808702:12): 2.7851638631 = = **1.0366919**	(128.4882898189:12) : 10.5146222424 = =**1.0018330**

Source: Realised by the author. *Note: Value of π = 3.1415926536 and of circumradius (r_c) = 1 or 100%

After finishing the geometric calculations in Table no. 6a, the statistical comparison of the two demographic PSIN values (VPSIN and APSIN based directly on the volume and surface indices in the dodecahedron) is carried out in Table no. 6b, starting from the degree of proximity to the general dynamics of the population in the EU between 2020 and 2021 (i.e. described in Table no. 6c).

Table no. 6b: Statistical confrontation of the demographic IN and PSIN (VPSIN & APSIN), based on direct indices of volume & area in dodecahedron

IN Classic calculus *Standard*	**VPSIN** - *Standard values IN =1 &*	**APSIN** - *Standard values IN =1 &*

value IN = 1	(r_c) = 1	(r_c) = 1
0.9786276	0.9372433760	0.9577119795
1.0041778	1.0125858350	1.0083730540
0.9969231	0.9907976728	0.9938556673
0.9762795	0.9305131397	0.9531216621
1.1760124	1.6264312232	1.3830051650
0.8570580	0.6295505958	0.7345484154
0.9907121	0.9723942941	0.9815104651
1.02	1.0612080000	1.0404000000
0.9855769	0.9573517771	0.9713618258
0.9657647	0.9007701416	0.9327014558
1.1273125	1.4326264592	1.2708334727
0.9962627	0.9888299500	0.9925393674
12.074707: 12= **1.0062256**	12.4403024645:12 = **1.0366919**	12.219962530:12 = **1.0018330**

*Source: Realised by the author. *Note: Value of* π *= 3.1415926536 and of circumradius* (r_c) *= 1 or 100%.*

*** Note: The major statistical advantage of PSIN (VPSIN and APSIN), visible both in Table 6b and in 6a, is offered by the extensive variability of the values of the new volume and area indices in this specific case of the dodecahedron used as Platonic Solid (PS).*

From the completely new result presented in Table no. 6a and 6b anyone can see that the original determination of the PSIN (VPSIN and APSIN) offers an interpretive framework with a greater potential, compared to the gross change index in classical IN calculus. When the research needs or demands a more performing PSIN (VPSIN or APSIN), the number of demographic factors can increase and the icosahedron may be used as a Platonic Solid with 20 equal surfaces or under the influence of 20 distinct factors, also extracted from the demographic family in this case (Table no. 7, 7a, 7b).

The increase in the number of factors practically amplifies the quality of the new PSIN as a geometric index of volume or surface and diminishes the gap compared to the classic index of the demographic evolution of the entire population. New values reveal that immigration and marriages have a slightly corrected impact on the ensemble of the final innovative construction.

Table no. 7: Some Major Demographical Factors (generating the 20 PSIN's key factors in EU)

Previous year = 1.000000

No	Key factor and derived Index Number	Value*
1.	Deaths (2020/2021)	0.9786276
2.	Life births (2021/2020)	1.0041778
3.	Net natural change (2021/2020)	0.9969231
4.	Infant mortality rate (2021/2020)	0.9762795
5.	*Immigration from outside EU* (2021/2020)	1.1760124
6.	Emigration to outside EU (2020/2021)	0.8570580
7.	Healthy life years at birth (2021/2020)	0.9907121
8.	Fertility rate (2021/2020)	1.02
9.	Ageing (65 years & more) 2020/2021	0.9855769
10.	Crude divorce rate (2020/2021)**	0.9657647
11.	*Crude marriage rate* (2021/2020)**	1.1273125
12.	Life expectancy at birth (2021/2020)	0.9962627
13.	Net migration (2021/2020)	1.0049068
	1.024 : 101.9 = 1.00490677	
14.	Healthy life years at birth - F (2021/2020)	0.9907834
	64.5 : 65.1 = 0.99078341	
15.	Healthy life years at birth - M (2021/2020)	0.9890966
	63.5 : 64.2 = 0.989096573	
16.	Urban population (2021/2020)	1.0020585
	336,283,387: 335,592,576 = 1.002058481	
17.	Total-age dependency ratio (2020/2021)	0.9910714
	0.555 : 0.560 = 0.991071428	
18.	Age Structure (15-64)/Total (2021/2020)	0.9953416
	0.641 :0.644 = 0.995341614	
19.	Age structure (≥65)/ Total (2020/2021)	0.9855769
	0.205 :0.208 = 0.985576923	
20.	Employment rate (2021/2020)	1.0195258
	0.731 : 0.717 = 1.019525802	

Source:https://ec.europa.eu/eurostat/databrowser/bookmark/e7fb6d0d-90e4-4024-a53d-30b15751c3bd?lang=en

*All of the demographic key factors' information is selected from https://chat.openai.com being analyzed by the author based on the criterion of a maximum of 12 dodecahedron's indicators, and thus becoming a narrow list of a more extensive one in relation to the data availability. The data construction of comparable indicators is based on both statistical and demographic standards. *Note: Each value is expressed as index numbers (IN) according to the positive impact of the key factor on the demographic evolution of the EU, dividing the indicator from 2021 to that of 2020 (i.e. births) or vice versa (i.e. deaths). **Note: 58.2 % of children are born inside marriages two corrections were needed to IN from lines 11-12.*

Table no. 7a: Statistical confrontation of the demographical IN and PSIN (VPSIN & APSIN), based on aggregates of volumes & areas, through a pragmatic calculation focused on the icosahedron

IN Classic calculus *Standard value IN =1*	**VPSIN**-*Standard values IN =1 & (r_c) = 1* $V=[(15+5\sqrt{5}):12]\times s^3$ = 2.5361507101	**APSIN** - *Standard values IN =1 & (r_c)=1* $S=[5\sqrt{3}\times s^2]$ = 9.5745413833
0.9786276	2.3769904535	9.1696529808
1.0041778	2.5680702844	9.6547095354
0.9969231	2.5128122215	9.5157122157
0.9762795	2.3599215601	9.1257027973
1.1760124	4.1248747018	13.2416401851
0.8570580	1.5966351905	7.0329642009
0.9907121	2.4661384793	9.3975125661
1.02	2.6913834228	9.9613528552
0.9855769	2.4279883892	9.3003439994
0.9657647	2.2844888343	8.9301886865
1.1273125	3.6333566117	12.1676476752
0.9962627	2.5078217800	9.5031092478
1.0049068	2.5736673490	9.6687326258
0.9907834	2.4666709700	9.3988652633
0.9890966	2.4540939499	9.3668895353
1.0020585	2.5518449712	9.6140003415
0.9910714	2.4688226243	9.4043301643
0.9953416	2.5008723490	9.4855450703
0.9855769	2.4279883892	9.3003439994
1.0195258	2.6876314793	9.9520929032
20.0530683:20 =**1.0026534**	**(51.682074011:20) : : 2.5361507101 = = 1.0189078**	(193.1913368485: 20):9.5745413833 = **1.0088804**

*Source: Realised by the author. *Note: Value of* π = *3.1415926536 and of circumradius* (r_c) = *1 or 100%*

Table no. 7b: Statistical confrontation of the demographical IN and PSIN (VPSIN & APSIN), based on direct indices of volume & area in the icosahedron

IN Classic calculus *Standard value IN = 1*	**VPSIN -** *Standard values IN =1 & (r_c) = 1*	**APSIN -** *Standard values IN =1 & (r_c) = 1*
0.9786276	0.9372433757	0.9577119715
1.0041778	1.0125858347	1.0083730456
0.9969231	0.9907976726	0.9938556591
0.9762795	0.9305131395	0.9531216542
1.1760124	1.6264312229	1.3830051535
0.8570580	0.6295505956	0.7345484093
0.9907121	0.9723942938	0.9815104569
1.02	1.0612079998	1.0403999914
0.9855769	0.9573517768	0.9713618178
0.9657647	0.9007701414	0.9327014480
1.1273125	1.4326264588	1.2708334621
0.9962627	0.9888299498	0.9925393592
1.0049068	1.0147927480	1.0098376683
0.9907834	0.9726042540	0.9816517376
0.9890966	0.9676451559	0.9783120760
1.0020585	1.0061882208	1.0041212291
0.9910714	0.9734526477	0.9822225117
0.9953416	0.9860898008	0.9907048925
0.9855769	0.9573517768	0.9713618178
1.0195258	1.0597286147	1.0394328482
20.0530683:20 = **1.0026534**	20.3781556801 : 20 = **1.0189078**	20.1776072098 : 20 = **1.0088804**

Source: Realised by the author. *Note: Value of π = 3.1415926536 and of circumradius (r_c) = 1 or 100%

All the 20 demographic indicators selected as factors or variables represent useful tools to measure, understand & describe the major characteristics of any human population with a real impact on population evolutions or involutions (i.e. deaths, life births and the real difference as net natural change, together with immigration from outside and emigration to outside for estimating the limits of population's size in the future).

But many other factors are implied in these estimations and the results are different if the factors changings are higher. Thus the impact of each factor and variable must be carefully analysed (i.e. in human population prediction the impact of fertility rate or marriage rate levels, are more than significant for any demographic estimation). In the end, which aggregate indicator as an index number will be chosen from the six indices, two classic and four of the VPSIN and APSIN type? As the author of these original statistical constructions, I believe in VPSIN and APSIN as new types of IN calculus, especially when the evolutions tend more and more to exponentiality and not linearity.

But among the four original constructions focused on the dodecahedron and the icosahedron, which PSIN is preferable to be chosen? For factors or variables, one focused on the icosahedron, and for beginnings, the one focused on facets (APSIN = **1.0088804**) and if an upward continuity or even an exaggeration is identified, VPSIN is the final solution (**1.0189078**).

5. CONCLUSIONS

RPH is a geometric solid if the next conditions are met: a) convexity (normal variability, neither explosion, nor implosion, in VPSIN or APSIN cases, the final PSIN remaining positive as values); b) symmetry (but in the area or volume images)10; c) identity of all surface polygons to each other (equality of factors as importance in VPSIN or APSIN cases of PSIN) d) identity of the dihedral angles (equality of radius as factors of the initial construction of VPSIN or APSIN cases of PSIN).

The constructive option initiated methodologically as PSIN (VPSIN or APSIN) after GAIN [6; 42-43]; and GIN [46] develops the idea from a simple circle and regular polygon area to a sphere and to the polyhedra volume. The originality of the general IN, defined as PSIN (VPSIN or APSIN) is not limited to just a few constructive options of this new concept, but it offers also a normal evolution, related to the history of IN and INM. Derived from GAIN [6], the original construction and methodology of PSIN (VPSIN or APSIN) opted for the regularly inscribed polyhedra in a sphere because it involves: (1) a careful analysis of the Platonic solids typology; (2) a reasonable statistical error level; and (3) a valid confrontation ensuring of a real degree of comparability between VPSIN and APSIN…

VPSIN is focused on volumes or (three) dimensionality and APSIN remains addicted to surfaces or (bi)dimensionality as the dominant way of thinking but also ensures temporal, territorial, and structural aspects. Addressing the research gap in the creative development of statistical confrontation the paper underlines the novel mathematical (geometrical) application, defines the research problems, and analyses why, the authors chose a particular issue to research on means, comparing various methods with their pros and cons, formulating logical premise and choosing the adequate hypothesis, and selecting the precise data source and collection to analyze and discuss the results. VPSIN creatively ensures an ascending and significant degree of coverage of simultaneous evolutions balanced/unbalanced by the number of factors and variables in parallel, with a limiting statistical error descending evolutionary… VPSIN and APSIN are practically and historically some coherent, constructive instruments and offer multidimensional confrontation, starting from various statistical databases chosen for the adequate field of microeconomic management…

Some future research can offer some useful statistical tables, based on geometric calculus for every 0.001 difference (or for each + or - 0,1%) or based on (in)sphere radius (r_i) median radius (r_m) and not only (circum)sphere radius (r_c), and especially an aggregate or PSIN case. A particularly interesting fact is the combination of PS subcomponents (4, 6, 8, 12 and 60, 20) that allow the construction of weighted indices with specific weighting coefficients (4 becoming 25% per component, 6 approximately 16.666%, 8 translating a structure of 12.5%, 12 about 8.333 % (or half of 6), 20 exactly 5%, and 60 approximately 1.666 or 1/10 of 6. In this case, sums can be generated and various equalities with 100% capitalizing on all forms of PS, to emphasize the different importance of the factors, calculating previous error for PSIN aggregated or weighted in this way [e.g. 100 % means two subcomponents of tetrahedron (two 4-hedron = 50%) plus two components of Hexahedron (two 6-hedron = 33.333%) plus 10 from 60 components of dodecahedron (ten 60-hedron = 16.666%)] etc.

6. SOME FINAL REMARKS

As new subdomais in scientific research, demo(gra)statistics and demographysics can combine better time projections of human populations' phenomena with much better calibrated extrapolations or interpolations in the economic, demographical, crime-related, electoral world. This aspect defines a real supremacy of statistical physics and modern statistical thinking incorporated by, and it appears at the very first moment as a better provider to the first practical substantial solutions, within the framework of the same subjects investigated by economics, sociology, demography, etc. The concrete forms of dissecting the thought of statistical thinking and statistical physics in the diversity of the real world redefine these new sciences such as econophysics, sociophysics, quantum economics, quantum, demo(gra)statistics, demographysics, etc. based on new instruments, techniques, methods and models considered to be more precisely and adequate [24]. In 2017, I tried in another paper, about population implosion to generate a model of cavitation applied to demography. The originality of that model was based on the hypothesis that in all states of flux (including demographic flux), as in all liquids, the phenomenon of cavitation appears when the local (or demographic) pressure decreases below the vaporization value (the demographic point of survival) respectively, under the saturation pressure (defined by the fertility rate), at the given temperature of gases (general demographic conditions), dissolved in the liquid [19].

Other new Index Numbers were created by me and exemplified in the economy (e.g. GAIN [6] to quantify the evolution of a complex economy such as England really is) or in tourist activity (e.g. [46], where the new GIN_V or GIN_A based on the volume and surface of minimal Platonic Solids focused on the pyramid and the cube are used to measure the real evolution, as accurately as possible, of this type of multidisciplinary activity, with multiple impacts.

In this paper, the new interdisciplinary science demographysics seems to have been anticipated by demo(gra)statistics and the specificity of the article's research offers a creative model of the geometric aggregate index number (PSIN), based on volume (VPSIN) or area (APSIN). From the methodological point of view, the new model insists on the need for a complex, more realistic and non-linear approach...

The instrumental future of PSIN is decided not only in relation to Euclidean geometries but can be adapted to non-Euclidean ones as well, which brings it closer to the universe of physics, sociophysics and econophysics. An aggregate tool of a holistic type, such as the statistical index, can be developed using weighting coefficients by simultaneously capitalizing on Platonic solids whose sections can reach and ensure the integrity of the whole (100%).

Within the next few years, I hope that this new instrument named PSIN (APSIN or VPSIN) will be expected to develop many demo(gra)statistical or demographysical methods in understanding human populations evolution processes generating new disciplines like demographysics, indexphysics or physicalprognosis, as result of new demographical (as essence), mathematical (as geometrical thought, especially), statistical (as instrumental solutions) and physical (as real and complex interactions) and even more or with many more classical sciences.

The most important and new domain, called demographysics (e.g. based on describing international migration) will need some special area of marketing and management, some distinctive aggregate index numbers (not as classical INs are, from poverty index to corruption or globalization index, from Consumer Price Index to Dow Jones Industrial Average, etc.) but most of all to prognosis or spatial and temporal estimation.

The accuracy of the new demo(gra)statistical or demographysical indices could be improved together by statistics and by statistical physics with their careful thinking in terms of geometrical or dimensional analysis, combined with better data analysis correlating variables as major factors to the phenomena, such as speeds, senses, orientations, spins, cords, for which many new index numbers (INs) are or can be designed.

The future of these new demo(gra)statistical or demographysical models, which start with GAIN, GIN and PSIN in an attempt to explain the mutations of birth and mortality, death and divorces, life expectancy and marriages, but more especially of fertility, ageing and migration, "*cannot maintain the old isolating and unidisciplinary mark, allowing access of the flourishing evolution of trans-, inter- and multidisciplinarity modelling*" and especially of a new holistic approach in scientific research [13].

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